

PROOF OF FORMULA 4.235.4

$$\int_0^1 \frac{x^{m-1} + x^{n-m-1}}{1-x^n} \ln x \, dx = -\frac{\pi^2}{n^2 \sin^2(\pi m/n)}$$

Start with

$$\int_0^1 \frac{x^{m-1} + x^{n-m-1}}{1-x^n} \ln x \, dx = \int_0^1 \frac{x^{m-1}}{1-x^n} \ln x \, dx + \int_0^1 \frac{x^{n-m-1}}{1-x^n} \ln x \, dx.$$

Entry **4.254.1** gives

$$\int_0^1 \frac{x^{p-1} \ln x}{1-x^q} \, dx = -\frac{1}{q^2} \psi' \left(\frac{p}{q} \right).$$

Therefore

$$\int_0^1 \frac{x^{m-1} + x^{n-m-1}}{1-x^n} \ln x \, dx = -\frac{1}{n^2} \left[\psi' \left(\frac{m}{n} \right) + \psi' \left(1 - \frac{m}{n} \right) \right].$$

The result follows from the relation

$$\psi'(a) + \psi'(1-a) = \frac{\pi^2}{\sin^2 \pi z}.$$