PROOF OF FORMULA 4.241.11

$$\int_0^1 \frac{\ln x \, dx}{\sqrt{x(1-x^2)}} = -\frac{\sqrt{2\pi}}{8} \left[\Gamma\left(\frac{1}{4} \right) \right]^2$$

In the proof of formula **4.253.1** it was shown that

$$\int_0^1 t^{a-1}(1-t)^{b-1} \ln t \, dt = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} \left[\psi(a) - \psi(a+b) \right].$$

The change of variable $t = x^2$ gives

$$\int_0^1 \frac{\ln x \, dx}{\sqrt{x(1-x^2)}} = \frac{1}{4} \int_0^1 t^{-3/4}(1-t)^{-1/2} \, \ln t \, dt.$$

It follows that

$$\int_0^1 \frac{\ln x \, dx}{\sqrt{x(1-x^2)}} = \frac{1}{4} \frac{\Gamma(1/4)\Gamma(1/2)}{\Gamma(3/4)} \left[\psi(1/4) - \psi(3/4) \right].$$

The identities

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x} \text{ and } \psi(x) - \psi(1-x) = -\pi \cot(\pi x),$$

is now used to produce the result.