PROOF OF FORMULA 4.241.9

$$\int_0^1 \sqrt{1 - x^2} \, \ln x \, dx = -\frac{\pi}{8}(1 + 2\ln 2)$$

Let $t = -\ln x$ to obtain
$$\int_0^1 \sqrt{1 - x^2} \, \ln x \, dx = -\int_0^\infty te^{-t}\sqrt{1 - e^{-2t}} \, dt.$$
This last integral is evaluated in **3.451.2**.