PROOF OF FORMULA 4.254.3

$$\int_0^\infty \frac{\ln x \, dx}{(x^q - 1)\, x^p} = \frac{\pi^2}{\left(q \, \sin\left(\frac{p-1}{q} \pi \right) \right)^2}$$

The change of variables $t = 1/x$ gives

$$\int_0^\infty \frac{\ln x \, dx}{(x^q - 1)\, x^p} = -\int_0^\infty \frac{t^{p+q-2} \, \ln t}{1 - t^q} \, dt.$$

The result now follows from entry **4.254.2**:

$$\int_0^\infty \frac{x^{a-1} \, \ln x}{1 - x^b} \, dx = -\frac{\pi^2}{b^2 \sin^2(\pi a/b)}.$$