PROOF OF FORMULA 4.257.1

$$\int_0^\infty \frac{x^{\nu}}{(x+b)(x+c)} \,\ln\left(\frac{x}{b}\right) \, dx = \frac{\pi}{(c-b)\sin\pi\nu} \left[c^{\nu}\ln\frac{c}{b} + \pi \left(b^{\nu} - c^{\nu}\right)\cot\pi\nu\right]$$

The change of variables x = bt gives

$$\int_0^\infty \frac{x^\nu}{(x+b)(x+c)} \, \ln\left(\frac{x}{b}\right) \, dx = b^{\nu-1} \int_0^\infty \frac{t^\nu \ln t}{(t+1)(t+c/b)} \, dt.$$

This last integral appears in entry 4.252.1 and the result follows from there.