PROOF OF FORMULA 4.261.17

$$\int_0^1 (\ln x)^2 \; x^{\mu-1}(1-x)^{\nu-1}\, dx = B(\mu,\nu)\left[(\psi(\mu)-\psi(\mu+\nu))^2+\psi'(\mu)-\psi'(\mu+\nu)\right]$$

Differentiate the relation

$$B(\mu,\nu) = \int_0^1 x^{\mu-1}(1-x)^{\nu-1}\, dx$$

twice with respect to the parameter μ. The first time gives

$$\frac{d}{d\mu}B(\mu,\nu) = B(\mu,\nu)\left[\psi(\mu)-\psi(\mu+\nu)\right].$$

This is equivalent to entry **4.253.1**. A second differentiation gives the stated result.