PROOF OF FORMULA 4.271.14

$$\int_0^\infty \frac{x^{\nu-1}\ln^n x\,dx}{x^2+2ax\cos t+a^2}\,dx = -\frac{\pi}{\sin t}\frac{d^n}{d\nu^n}\left[a^{\nu-2}\frac{\sin(\nu-1)t}{\sin\pi\nu}\right]$$

Let

$$f(a) = \int_0^\infty \frac{x^{\nu-1}\,dx}{x^2+2ax\cos t+a^2}.$$

The scaling $x = ay$ yields

$$f(a) = a^{\nu-2}\int_0^\infty \frac{y^{\nu-1}\,dy}{y^2+2y\cos t+1}.$$

Factor

$$y^2+2y\cos t+1 = (y+\alpha)(y+\beta)$$

with $\alpha = -e^{-it}$ and $\beta = -e^{it}$.

Entry **3.223.1** gives

$$\int_0^\infty \frac{x^{\nu-1}\,dx}{(x+\alpha)(x+\beta)} = \frac{\pi}{\alpha-\beta}\left(\beta^{\nu-1}-\alpha^{\nu-1}\right)\ \text{cosec}\ (\pi\nu).$$

The values of α and β now give

$$\int_0^\infty \frac{x^{\nu-1}\,dx}{(x+\alpha)(x+\beta)} = -\frac{\pi}{\sin t}\frac{\sin((\nu-1)t)}{\sin\pi\nu}.$$

The result follows by differentiating n times with respect to ν.