PROOF OF FORMULA 4.271.6

$$\int_0^1 \frac{(\ln x)^{2n}\, dx}{1+x^2} = \frac{1}{2}\int_0^\infty \frac{(\ln x)^{2n}\, dx}{1+x^2} = \frac{\pi^{2n+1}}{2^{2n+2}}|E_{2n}|$$

Split the integral at $x = 1$ and make the change $x \mapsto 1/x$ to see the first identity. Entry **4.271.5** states that

$$\int_0^1 \frac{(\ln x)^{2n}\, dx}{1+x^2} = (2n)! \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{2n+1}}.$$

The result follows from the expression of this series in terms of the Euler numbers

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{2n+1}} = \frac{\pi^{2n+1}\,|E_{2n}|}{(2n)!\, 2^{2n+2}}.$$