

PROOF OF FORMULA 4.272.5

$$\int_1^\infty \frac{\ln^p x}{x^2} dx = \Gamma(1 + p)$$

Let $t = 1/x$ to obtain

$$\int_1^\infty \frac{\ln^p x}{x^2} dx = \int_0^1 (-\ln t)^p dt.$$

The change of variables $z = -\ln t$ gives

$$\int_0^1 (-\ln t)^p dt = \int_0^\infty z^p e^{-z} dz.$$

This is the integral representation for $\Gamma(1 + p)$.