

Calculus 121: Take Home Test

- 1) Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.
- 2) Let $f(x) = x^2 + x$. Find all possible tangents to $f(x)$ that contain the point $(2, -3)$. What happens for $(2, 7)$? Explain.
- 3) How many normal lines to $y = x^2$ pass through the point $(0, c)$?
- 4) Suppose that $f(5) = 1$, $f'(5) = 10$, $g(5) = 2$ and $g'(5) = 3$. Find $(fg)'(5)$, $(f/g)'(5)$ and $(g/f^2)'(5)$.
- 5) If f and g have derivatives of all orders and $h(x) = f(x)g(x)$ make a guess for the n -th derivative of h in terms of those of f and g . Start with small values of n .
- 6) Prove that the derivative of $f(x) = \sec x$ is $\sec x \tan x$.
- 7) If $h(x) = f(g(x))$ and $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$ and $g'(5) = 6$. Find $h'(5)$.
- 8) Let $f(x) = xg(x^2)$ for a given function g . Find an expression for the first three derivatives of f .
- 9) Let $f(x) = Ae^{-x} + Bxe^{-x}$. Evaluate $f'' + 2f' + f$.
- 10) If an equation of motion of a particle is given by $s(t) = A \cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion*. Find the velocity of the particle at time t . When is the velocity zero? Make a graph where the horizontal axis has the position $s(t)$ and the vertical axis has the velocity. Describe the motion of the particle in this axes.
- 11) Show that the function $y = e^{rx}$ satisfies the equation $y'' + 5y' - 6y = 0$ for some specific values of r . How do you find these r ? Use this result to solve the differential equation $y'' - 3y' + 2y = 0$.
- 12) Find the derivative of $y = y(x)$ by implicit differentiation if $e^{y/x} = x - y$.
- 13) Find the equation of the tangent line to the curve defined implicitly by $y^2(y^2 - 4) = x^2(x^2 - 5)$ at the points where $y = 0$.
- 14) Show that the sum of the x and y intercepts of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c .
- 15) Find the derivative of $f(x) = (1 + \ln x)^2 / (1 - \ln x)^3$.
- 16) Find the derivative of $f(x) = x^{\sin x} + (\sin x)^x$.
- 17) Find the n -th derivative of the function $h(x) = x^n / (1 - x)$.
- 18) Find points on the curve $y(x) = x^4 - 2x^2 - x$ that have a common tangent line.