

PROOF OF FORMULA 2.315

$$\int \frac{dx}{\sqrt{a+be^{mx}}} = \begin{cases} \frac{1}{m\sqrt{a}} \ln \left(\frac{\sqrt{a+be^{mx}}-\sqrt{a}}{\sqrt{a+be^{mx}}+\sqrt{a}} \right) & \text{if } a > 0 \\ \frac{2}{m\sqrt{-a}} \tan^{-1} \left(\frac{\sqrt{a+be^{mx}}}{\sqrt{-a}} \right) & \text{if } a < 0 \end{cases}$$

Let $t = \sqrt{a+be^{mx}}$ to produce

$$\int \frac{dx}{\sqrt{a+be^{mx}}} = \frac{2}{m} \int \frac{dt}{t^2-a}.$$

The result is now elementary.