PROOF OF FORMULA 2.325.1

$$\int \frac{e^{ax}}{x} dx = \mathrm{Ei}(ax)$$

The exponential integral is defined by

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t}\, dt$$

for $x < 0$ and by its principal value for $x > 0$. Formula 2.324.2 has the expression

$$\int \frac{e^{ax}\, dx}{x^m} = -\frac{e^{ax}}{(m-1)!} \sum_{k=1}^{m-1} \frac{a^{k-1}\,(m-k-1)!}{x^{m-k}} + \frac{a^{m-1}}{(m-1)!}\mathrm{Ei}(ax).$$

This formula, with $m = 1$, gives the present one. To establish it directly, let $t = ax$.