

PROOF OF FORMULA 2.325.3

$$\int \frac{e^{ax}}{x^3} dx = -\frac{e^{ax}}{2x^2} - \frac{ae^{ax}}{2x} + \frac{a^2}{2}\text{Ei}(ax)$$

The exponential integral is defined by

$$\text{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} dt$$

for $x < 0$ and by its principal value for $x > 0$. Formula 2.324.2 has the expression

$$\int \frac{e^{ax} dx}{x^m} = -\frac{e^{ax}}{(m-1)!} \sum_{k=1}^{m-1} \frac{a^{k-1} (m-k-1)!}{x^{m-k}} + \frac{a^{m-1}}{(m-1)!} \text{Ei}(ax).$$

This formula, with $m = 3$, gives the present one.