

PROOF OF FORMULA 2.33.1

$$\int e^{-(ax^2+2bx+c)} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} e^{(b^2-ac)/a} \operatorname{erf}\left(\sqrt{a}x + \frac{b}{\sqrt{a}}\right)$$

Complete the square to obtain

$$ax^2 + 2bx + c = a\left(x + \frac{b}{a}\right)^2 + \frac{ac - b^2}{a}.$$

The change of variables $u = x + b/a$ gives

$$\int e^{-(ax^2+2bx+c)} dx = e^{(b^2-ac)/a} \int e^{-au^2} du.$$

Now let $v = \sqrt{a}u$ to produce

$$\int e^{-(ax^2+2bx+c)} dx = \frac{1}{\sqrt{a}} e^{(b^2-ac)/a} \int e^{-v^2} dv.$$

The result follows from the definition of the error function

$$\operatorname{erf}(v) = \frac{2}{\sqrt{\pi}} \int e^{-v^2} dv.$$