

PROOF OF FORMULA 3.191.2

$$\int_a^\infty x^{-\nu}(x-a)^{\mu-1} dx = a^{\mu-\nu} B(\nu-\mu, \mu)$$

Let $x = at$ to obtain

$$\int_a^\infty x^{-\nu}(x-a)^{\mu-1} dx = a^{\mu-\nu} \int_1^\infty t^{-\nu}(t-1)^{\mu-1} dt.$$

The change of variables $s = 1/(t-1)$ produces

$$a^{\mu-\nu} \int_1^\infty t^{-\nu}(t-1)^{\mu-1} dt = a^{\mu-\nu} \int_0^\infty \frac{s^{\nu-\mu-1} ds}{(1+s)^\nu}.$$

The result now follows from the integral representation

$$B(a, b) = \int_0^\infty \frac{t^{a-1} dt}{(1+t)^{a+b}}.$$