

PROOF OF FORMULA 3.193

$$\int_0^n x^{\nu-1}(n-x)^n dx = \frac{n^{\nu+n} n!}{(\nu)_{n+1}} = \frac{n^{\nu+n} n!}{\nu(\nu+1)\cdots(\nu+n)}$$

Let $x = nt$ to obtain

$$\int_0^n x^{\nu-1}(n-x)^n dx = n^{\nu+n} \int_0^1 t^{\nu-1}(1-t)^n dt.$$

The integral representation

$$B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$$

shows that the required integral is

$$B(\nu, n+1) = \frac{\Gamma(\nu)\Gamma(n+1)}{\Gamma(n+1+\nu)},$$

and the result follows from the relation

$$(a)_m = \frac{\Gamma(a+m)}{\Gamma(a)},$$

among the gamma function and the Pochhammer symbol

$$(a)_m = a(a+1)\cdots(a+m-1).$$