PROOF OF FORMULA 3.194.1

$$\int_0^a \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = \frac{a^\mu}{\mu}\,{}_2F_1\left[\nu,\mu;\mu+1;-ab\right]$$

The proof employs the integral representation

$${}_2F_1\left[\alpha,\beta;\gamma;z\right] = \frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}\,dt.$$

Let $x = at$ to obtain

$$\int_0^a \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = a^\mu \int_0^1 t^{\mu-1}(1+abz)^{-\nu}\,dt.$$

Therefore, choose $\alpha=\nu$, $\beta=\mu$, $\gamma = 1+\mu$ and $z=-ab$ to obtain

$$\int_0^a \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = a^\mu B(\mu,1)\,{}_2F_1\left[\nu,\mu;1+\mu;-ab\right].$$

The result is simplified by using $B(\mu,1) = 1/\mu$.