

PROOF OF FORMULA 3.194.7

$$\int_0^\infty \frac{x^m dx}{(a+bx)^{n+1/2}} = 2^{m+1} m! \frac{(2n-2m-3)!!}{(2n-1)!!} \frac{a^{m-n+1/2}}{b^{m+1}}$$

Let $x = at/b$ to produce

$$\int_0^\infty \frac{x^m dx}{(a+bx)^{n+1/2}} = \frac{a^{m-n+1/2}}{b^{m+1}} \int_0^\infty \frac{t^m dt}{(1+t)^{n+1/2}}.$$

The integral representation

$$B(u, v) = \int_0^\infty \frac{t^{u-1} dt}{(1+t)^{u+v}},$$

shows that

$$\int_0^\infty \frac{t^m dt}{(1+t)^{n+1/2}} = B(m+1, n-m-1/2).$$

The result now follows from

$$\Gamma(r + \frac{1}{2}) = \frac{\sqrt{\pi} (2r-1)!!}{2^r}.$$