

PROOF OF FORMULA 3.196.2

$$\int_a^\infty (x+b)^{-\nu}(x-a)^{\mu-1} dx = (a+b)^{\mu-\nu} B(\nu-\mu, \mu)$$

Let $t = a/x$ to obtain

$$\int_a^\infty (x+b)^{-\nu}(x-a)^{\mu-1} dx = a^{\mu-\nu} \int_0^1 t^{\nu-\mu-1}(1-t)^{\mu-1}(1+bt/a)^{-\nu} dt.$$

The integral representation of the hypergeometric function

$${}_2F_1[\alpha, \beta; \gamma; z] = \frac{1}{B(\beta, \gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha} dt,$$

shows that

$$\int_a^\infty (x+b)^{-\nu}(x-a)^{\mu-1} dx = a^{\mu-\nu} B(\nu-\mu, \mu) {}_2F_1[\nu, \nu-\mu; \nu; -b/a].$$

The result now follows from the identity

$$(1+z)^a = {}_2F_1[-a, s; s; -z].$$