PROOF OF FORMULA 3.196.4

$$\int_1^\infty \frac{dx}{(a-bx)(x-1)^\nu} = -\frac{\pi}{b\sin\pi\nu}(1-a/b)^{-\nu}$$

Let $t=x-1$ to obtain

$$\int_1^\infty \frac{dx}{(a-bx)(x-1)^\nu} = \int_0^\infty \frac{dt}{[(a-b)-bt]\; t^\nu}.$$

The change of variables $t=(b-a)s/b$ gives

$$\int_0^\infty \frac{dt}{[(a-b)-bt]\; t^\nu} = -\frac{1}{b}(1-a/b)^{-\nu}\int_0^\infty \frac{ds}{(1+s)\, s^\nu}.$$

The integral representation

$$B(u,v) = \int_0^\infty \frac{s^{u-1}\, ds}{(1+s)^{u+v}},$$

shows that

$$\int_0^\infty \frac{ds}{(1+s)\, s^\nu} = B(1-\nu,\nu).$$

The result follows from the identity

$$B(1-\nu,\nu) = \frac{\pi}{\sin\pi\nu}.$$