PROOF OF FORMULA 3.197.1

$$\int_0^\infty x^{\nu-1}(x+a)^{-\mu}(x+b)^{-\rho}\,dx = a^{-\mu}b^{\nu-\rho}B(\nu,\mu-\nu+\rho){}_2F_1\left[\mu,\nu;\mu+\rho;1-b/a\right]$$

Let $t = \frac{x}{x+b}$ to obtain

$$\int_0^\infty x^{\nu-1}(x+a)^{-\mu}(x+b)^{-\rho}\,dx = b^{\nu-\rho}a^{-\mu}\int_0^1 t^{\nu-1}(1-t)^{\mu+\rho-\nu-1}\left[1-(1-a/b)t\right]^{-\mu}\,dt.$$

The result now follows from the integral representation of the hypergeometric function

$${}_2F_1\left[\alpha,\beta;\gamma;z\right] = \frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}\,dt.$$