

PROOF OF FORMULA 3.197.3

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-bx)^{-\nu} dx = B(\lambda, \mu) {}_2F_1[\nu, \lambda; \lambda + \nu; b]$$

The basic integral representation for the hypergeometric function is

$${}_2F_1[\alpha, \beta; \gamma; z] = \frac{1}{B(\beta, \gamma - \beta)} \int_0^1 x^{\beta-1}(1-x)^{\gamma-\beta-1}(1-zx)^{-\alpha} dx.$$

Replace $\beta \mapsto \lambda$, $\alpha \mapsto \nu$, $\gamma \mapsto \mu + \lambda$ and $z \mapsto b$ to get the result.