PROOF OF FORMULA 3.197.8

$$\int_0^a x^{\nu-1}(x+b)^{\lambda}(a-x)^{\mu-1}dx = b^{\lambda}a^{\mu+\nu-1}B(\mu,\nu){}_2F_1\left[-\lambda,\nu;\mu+\nu,-\tfrac{a}{b}\right]$$

Let $x = at$ to obtain

$$\int_0^a x^{\nu-1}(x+b)^{\lambda}(a-x)^{\mu-1}dx = a^{\mu+\nu-1}b^{\lambda}\int_0^1 t^{\nu-1}(1-t)^{\mu-1}(1+at/b)^{\lambda}.$$

The result now comes from the integral representation of the hypergeometric function

$${}_2F_1[\alpha,\beta;\gamma;z] = \frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-1}(1-tz)^{-\alpha}\,dt.$$