PROOF OF FORMULA 3.223.2

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(x+b)(a-x)} = \frac{\pi}{(a+b)\,\sin \pi\mu}\left(b^{\mu-1} + a^{\mu-1}\cos \mu\pi\right)$$

The partial fraction decomposition

$$\frac{1}{(x+b)(a-x)} = \frac{1}{a+b}\left(\frac{1}{b+x} + \frac{1}{a-x}\right)$$

gives

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(x+b)(a-x)} = \frac{1}{a+b}\int_0^\infty \frac{x^{\mu-1}\,dx}{x+b} + \frac{1}{a+b}\int_0^\infty \frac{x^{\mu-1}\,dx}{a-x}.$$

Introduce the changes of variables $x = bt$ and $x = at$ in the first and second integral, respectively, to obtain

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(x+b)(a-x)} = \frac{b^{\mu-1}}{a+b}\int_0^\infty \frac{t^{\mu-1}\,dt}{t+1} + \frac{a^{\mu-1}}{a+b}\int_0^\infty \frac{t^{\mu-1}\,dt}{1-t}.$$

These integrals have been evaluated in 3.222.2. Their values are

$$\int_0^\infty \frac{t^{\mu-1}\,dt}{t+1} = \frac{\pi}{\sin \pi\mu}$$

and

$$\int_0^\infty \frac{t^{\mu-1}\,dt}{1-t} = \frac{\pi\,\cos \pi\mu}{\sin \pi\mu}.$$

This gives the result.