

**PROOF OF FORMULA 3.225.2**

$$\int_1^\infty \frac{(x-1)^{1-p}}{x^3} dx = \frac{p(1-p)\pi}{2 \sin \pi p}$$

Let  $t = 1/x$  to obtain

$$\int_1^\infty \frac{(x-1)^{1-p}}{x^3} dx = \int_0^1 t^p (1-t)^{1-p} dt.$$

The integral representation

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt$$

shows that the last integral is

$$B(p+1, 2-p) = \frac{\Gamma(p+1)\Gamma(2-p)}{\Gamma(3)}.$$

The identities  $\Gamma(a+1) = a\Gamma(a)$  and  $\Gamma(a)\Gamma(1-a) = \pi/\sin \pi a$  simplify the answer.