

PROOF OF FORMULA 3.226.1

$$\int_0^1 \frac{x^n dx}{\sqrt{1-x}} = \frac{2(2n)!!}{(2n+1)!!}$$

The integral representation

$$B(a, b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx$$

gives

$$\begin{aligned} \int_0^1 \frac{x^n dx}{\sqrt{1-x}} &= B\left(n+1, \frac{1}{2}\right) \\ &= \frac{\Gamma(n+1)\Gamma(1/2)}{\Gamma\left(n+\frac{3}{2}\right)}. \end{aligned}$$

The result follows from

$$\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{m-1}}(2m-1)!!$$