

FORMULA 3.231.5

$$\int_0^1 \frac{x^{\mu-1} - x^{\nu-1}}{1-x} dx = \psi(\nu) - \psi(\mu)$$

The integral representation

$$\psi(a) = - \int_0^1 \left(\frac{1}{\ln x} + \frac{x^{a-1}}{1-x} \right) dx,$$

is given as 8.361.4.

Then

$$\int_0^1 \frac{x^{\mu-1} - x^{\nu-1}}{1-x} dx = - \int_0^1 \left(\frac{1}{\ln x} + \frac{x^{\nu-1}}{1-x} \right) dx + \int_0^1 \left(\frac{1}{\ln x} + \frac{x^{\mu-1}}{1-x} \right) dx$$

gives the result.