PROOF OF FORMULA 3.231.6

$$\int_0^\infty \frac{x^{p-1} - x^{q-1}}{1-x}\, dx = \pi \left(\cot \pi p - \cot \pi q\right)$$

The change of variables $x \mapsto 1/x$ in the integral from 1 to ∞ yields

$$\begin{aligned}\int_0^\infty \frac{x^{p-1} - x^{q-1}}{1-x}\, dx &= \int_0^1 \frac{x^{p-1} - x^{q-1}}{1-x}\, dx - \int_0^1 \frac{x^{-p} - x^{-q}}{1-x}\, dx \\ &= \psi(q) - \psi(p) - [\psi(1-q) - \psi(1-p)] \\ &= [\psi(q) - \psi(1-q)] - [\psi(p) - \psi(1-p)]\end{aligned}$$

and the result follows from $\psi(x) - \psi(1-x) = \pi \cot \pi x$.