PROOF OF FORMULA 3.234.2

$$\int_0^1 \left(\frac{x^{q-1}}{1+ax} + \frac{x^{-q}}{a+x} \right) dx = \frac{\pi}{a^q \sin \pi q}$$

The change of variables $t = ax$ in the first integral and $x = at$ in the second one show that

$$\int_0^1 \left(\frac{x^{q-1}}{1+ax} + \frac{x^{-q}}{a+x} \right) dx = a^{-q} \left(\int_0^a \frac{t^{q-1}\,dt}{1+t} + \int_0^{1/a} \frac{t^{-q}\,dt}{1+t} \right).$$

Differentiation with respect to the parameter a shows that the sum of the two integrals is independent of a. Therefore

$$\int_0^1 \left(\frac{x^{q-1}}{1+ax} + \frac{x^{-q}}{a+x} \right) dx = a^{-q} \int_0^1 \frac{t^{q-1} + t^{-q}}{1+t}\,dt.$$

This integral is $\pi/\sin \pi q$ as shown in formula 3.231.2.