

PROOF OF FORMULA 3.235

$$\int_0^{\infty} \frac{(1+x)^{\mu} - 1}{(1+x)^{\nu}} \frac{dx}{x} = \psi(\nu) - \psi(\nu - \mu)$$

Formula 3.233 states that

$$\psi(\nu) = \int_0^{\infty} \left(\frac{1}{1+x} - \frac{1}{(1+x)^{\nu}} \right) \frac{dx}{x} - \gamma.$$

The result now follows from

$$\int_0^{\infty} \frac{(1+x)^{\mu} - 1}{(1+x)^{\nu}} \frac{dx}{x} = \int_0^{\infty} \left(\frac{1}{1+x} - \frac{1}{(1+x)^{\nu}} \right) \frac{dx}{x} - \int_0^{\infty} \left(\frac{1}{1+x} - \frac{1}{(1+x)^{\nu-\mu}} \right) \frac{dx}{x}.$$