PROOF OF FORMULA 3.241.4

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(p+qx^\nu)^{n+1}} = \frac{1}{\nu p^{n+1}} \left(\frac{p}{q}\right)^{\mu/\nu} \frac{\Gamma(\mu/\nu)\,\Gamma(n+1-\mu/\nu)}{\Gamma(n+1)}$$

Let $t = x^\nu$ to obtain

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(p+qx^\nu)^{n+1}} = \frac{1}{\nu}\int_0^\infty \frac{t^{\mu/\nu-1}\,dt}{(p+qt)^{n+1}}.$$

The change of variables $t = ps/q$ gives

$$\frac{1}{\nu}\int_0^\infty \frac{t^{\mu/\nu-1}\,dt}{(p+qt)^{n+1}} = \frac{1}{\nu}\left(\frac{p}{q}\right)^{\mu/\nu} \frac{1}{p^{n+1}} \int_0^\infty \frac{s^{\mu/\nu-1}\,ds}{(1+s)^{n+1}}.$$

The integral representation

$$B(a,b) = \int_0^\infty \frac{s^{a-1}\,ds}{(1+s)^{a+b}},$$

shows that the last integral is $B(\mu/\nu, 1-\mu/\nu+n)$. This is the stated result.