PROOF OF FORMULA 3.242.1

$$\int_{-\infty}^{\infty} \frac{x^{2m}\,dx}{x^{4n} + 2x^{2n}\cos t + 1} = \frac{\pi}{n} \sin\left[\frac{(2n-2m-1)t}{2n}\right] \text{ cosec } t \text{ cosec}\left(\frac{(2m+1)\pi}{2n}\right)$$

Assume $0 < m < n$ are integers and let $0 < s < 1$. The basic properties of the beta and gamma function give

$$B(s, 1-s) = \int_0^1 y^{-s}(1-y)^{s-1}\,dy = \int_0^\infty \frac{x^{s-1}\,dx}{x+1} = \frac{\pi}{\sin \pi s}.$$

Consequently, for $a > 0$,

$$\int_0^\infty \frac{x^{s-1}\,dx}{x+a} = \frac{\pi\, a^{s-1}}{\sin \pi s}.$$

Therefore

$$\int_0^\infty x^{s-1}\left[\frac{1}{x+e^{-t}} - \frac{1}{x+e^{t}}\right]\,dx = 2\pi \frac{\sinh(1-s)t}{\sin \pi s}.$$

That is

$$\int_0^\infty \frac{u^{s-1}\,du}{u^2 + 2u\cosh t + 1} = \frac{\pi\, \sinh(1-s)t}{\sin \pi s\, \sinh t}.$$

Now let $u = x^{2n}$, replace t by it and $s = (2m+1)/2n$. The result follows by observing that the integrand is even, so the integral over $[0, \infty)$ is half of that over the whole real line.