PROOF OF FORMULA 3.244.1

$$\int_0^1 \frac{x^{p-1}+x^{q-p-1}}{1+x^q}\,dx = \frac{\pi}{q\,\sin(\pi p/q)}$$

Let $t = x^q$ to obtain

$$\int_0^1 \frac{x^{p-1}+x^{q-p-1}}{1+x^q}\,dx = \frac{1}{q}\int_0^1 \frac{t^{p/q-1}+t^{-p/q}}{1+t}\,dt.$$

The function

$$\beta(a) = \int_0^1 \frac{t^{a-1}\,dt}{1+t},$$

is also given by

$$\beta(a) = \tfrac{1}{2}\left[\psi\left(\tfrac{a+1}{2}\right) - \psi\left(\tfrac{a}{2}\right)\right]$$

therefore

$$\int_0^1 \frac{x^{p-1}+x^{q-p-1}}{1+x^q}\,dx = \frac{1}{q}\left(\beta(\tfrac{p}{q}) + \beta(1-\tfrac{p}{q})\right).$$

This is now simplified using $\psi(1-x) - \psi(x) = \pi\cot\pi x$.