# PROOF OF FORMULA 3.249.4

$$\int_0^1 \frac{x^\mu \, dx}{1+x^2} = \frac{1}{2}\beta\left(\frac{\mu+1}{2}\right)$$

The change of variables $t = x^2$ gives

$$\int_0^1 \frac{x^\mu \, dx}{1+x^2} = \frac{1}{2}\int_0^1 \frac{t^{(\mu-1)/2} \, dt}{1+t}.$$

The result now follows from the integral representation

$$\beta(a) = \int_0^1 \frac{t^{a-1} \, dt}{1+t}.$$

This *beta function* can be given in terms of the polygamma function $\psi(x)$ as

$$\beta(a) = \frac{1}{2}\left(\psi\left(\frac{a+1}{2}\right) - \psi\left(\frac{a}{2}\right)\right).$$