PROOF OF FORMULA 3.249.5

$$\int_0^1 (1-x^2)^{\mu-1}\,dx = 2^{2\mu-2}B(\mu,\mu) = \tfrac{1}{2}B(\tfrac{1}{2},\mu)$$

Let $t = x^2$ to obtain

$$\int_0^1 (1-x^2)^{\mu-1}\,dx = \frac{1}{2}\int_0^1 (1-t)^{\mu-1}t^{-1/2}\,dt.$$

Using the integral representation of the beta function

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt,$$

we obtain

$$\int_0^1 (1-x^2)^{\mu-1}\,dx = \tfrac{1}{2}B(\tfrac{1}{2},\mu).$$

The second expression follows from the duplication formula for the gamma function

$$\Gamma(2a) = \frac{2^{2a-1}}{\sqrt{\pi}}\Gamma(a)\Gamma(a+\tfrac{1}{2}).$$

This appears as formula 8.335.1 in the table.