

PROOF OF FORMULA 3.251.10

$$\int_0^1 x^{p-1}(1-x^q)^{-p/q} dx = \frac{\pi}{q \sin(\pi p/q)}$$

Let $t = x^q$ to obtain

$$\begin{aligned} \int_0^1 x^{p-1}(1-x^q)^{-p/q} dx &= \frac{1}{q} \int_0^1 t^{p/q-1}(1-t)^{-p/q} dt \\ &= \frac{1}{q} B\left(\frac{p}{q}, 1 - \frac{p}{q}\right), \end{aligned}$$

using the integral representation for the beta function

$$B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt.$$

The result is simplified using the identity

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and

$$\Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$