PROOF OF FORMULA 3.251.5

$$\int_0^\infty \frac{x^{2m+1}\,dx}{(ax^2+c)^n} = \frac{m!\,(n-m-2)!}{2(n-1)!a^{m+1}c^{n-m-1}}$$

Let $x = \sqrt{c}t/\sqrt{a}$ so that $ax^2 = ct^2$ to obtain

$$\int_0^\infty \frac{x^{2m}\,dx}{(ax^2+c)^n} = \frac{c^{m+1-n}}{a^{m+1}} J(n,m),$$

where

$$J(n,m) = \int_0^\infty \frac{t^{2m+1}\,dt}{(1+t^2)^n}.$$

To evaluate this integral, let $u = t^2$ to obtain

$$J(n,m) = \frac{1}{2}\int_0^\infty \frac{u^m\,du}{(1+u)^n} = \frac{1}{2}B\left(m+1, n-m-1\right),$$

using the integral representation 8.380.3 in the table

$$B(a,b) = \int_0^\infty \frac{t^{a-1}\,dt}{(1+t)^{a+b}}.$$

Conclude that

$$J(n,m) = \frac{\Gamma(m+1)\,\Gamma(n-m-1)}{2\Gamma(n)} = \frac{m!\,(n-m-2)!}{2(n-1)!}.$$