PROOF OF FORMULA 3.251.7

$$\int_0^1 \frac{x^\mu\,dx}{(1+x^2)^2} = -\frac{1}{4} + \frac{(\mu-1)}{4}\beta\left(\frac{\mu-1}{2}\right)$$

The β-function has the integral representation

$$\beta(a) = \int_0^1 \frac{t^{a-1}\,dt}{1+t}.$$

The change of variables $t = x^2$ gives

$$\int_0^1 \frac{x^\mu\,dx}{(1+x^2)^2} = \frac{1}{2}\int_0^1 \frac{t^{(\mu-1)/2}\,dt}{(1+t)^2}.$$

The identity

$$\frac{1}{2}\int_0^1 \frac{t^{(\mu-1)/2}\,dt}{(1+t)^2} = -\frac{1}{2}\int_0^1 t^{(\mu-1)/2}\frac{d}{dt}\frac{1}{1+t}\,dt,$$

and integration by parts, gives the result.