PROOF OF FORMULA 3.267.1

$$\int_0^1 \frac{x^{3n}\,dx}{\sqrt[3]{1-x^3}} = \frac{2\pi}{3\sqrt{3}} \frac{\Gamma(n+\frac{1}{3})}{\Gamma(\frac{1}{3})\Gamma(n+1)} = \frac{2\pi}{3\sqrt{3}} \frac{\left(\frac{1}{3}\right)_n}{n!}$$

Let $t = x^3$ to obtain

$$\int_0^1 \frac{x^{3n}\,dx}{\sqrt[3]{1-x^3}} = \frac{1}{3}\int_0^1 \frac{t^{2n-3}\,dt}{(1-t)^{1/3}}.$$

The integral representation

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt$$

gives the last integral as

$$B\left(n+\tfrac{1}{3}, \tfrac{2}{3}\right) = \frac{\Gamma(n+\frac{1}{3})\Gamma(\frac{2}{3})}{\Gamma(n+1)}.$$

The result is simplified using

$$\Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$