PROOF OF FORMULA 3.268.2

$$\int_0^1 \frac{1-x^\mu}{1-x}\cdot x^{\nu-1}\,dx = \psi(\mu+\nu)-\psi(\nu)$$

The integral representation

$$\psi(a) = -\int_0^1 \left(\frac{1}{\ln t}+\frac{t^{a-1}}{1-t}\right)\,dt$$

gives

$$\begin{aligned}\int_0^1 \frac{1-x^\mu}{1-x}\cdot x^{\nu-1}\,dx &= -\int_0^1 \left(\frac{1}{\ln x}+\frac{x^{\mu+\nu-1}}{1-x}\right)\,dx \\ &+ \int_0^1 \left(\frac{1}{\ln x}+\frac{x^{\nu-1}}{1-x}\right)\,dx\end{aligned}$$

and this evaluates to $\psi(\mu+\nu)-\psi(\nu)$.