

PROOF OF FORMULA 3.269.2

$$\int_0^1 \frac{x^p - x^{-p}}{1+x^2} dx = \frac{1}{p} - \frac{\pi}{2} \operatorname{cosec} \left(\frac{p\pi}{2} \right)$$

The change of variable $t = x^2$ produces

$$\int_0^1 \frac{x^p - x^{-p}}{1+x^2} dx = \frac{1}{2} \int_0^1 \frac{t^{p/2} - t^{-p/2}}{1+t} dt.$$

The result now follows from entry 3.231.4 which states that

$$\int_0^1 \frac{t^a - t^{-a}}{1+t} dt = \frac{1}{a} - \frac{\pi}{\sin \pi a}.$$