

PROOF OF FORMULA 3.269.3

$$\int_0^1 \frac{x^\mu - x^\nu}{1 - x^2} dx = \frac{1}{2} \psi\left(\frac{\nu+1}{2}\right) - \frac{1}{2} \psi\left(\frac{\mu+1}{2}\right)$$

The change of variable $t = x^2$ yields

$$\int_0^1 \frac{x^\mu - x^\nu}{1 - x^2} dx = \frac{1}{2} \int_0^1 \frac{t^{\mu/2-1/2} - t^{\nu/2-1/2}}{1 - t} dt.$$

The result now follows from entry 3.231.5 which states

$$\int_0^1 \frac{x^{a-1} - x^{b-1}}{1 - x} dx = \psi(b) - \psi(a).$$