

**PROOF OF FORMULA 3.272.1**

$$\int_0^1 \frac{x^{n-1} + x^{n-1/2} - 2x^{2n-1}}{1-x} dx = 2 \ln 2$$

Write the integral as

$$\int_0^1 \frac{x^{n-1} + x^{n-1/2} - 2x^{2n-1}}{1-x} dx = \int_0^1 \frac{x^{n-1} - x^{2n-1}}{1-x} dx + \int_0^1 \frac{x^{n-1/2} - x^{2n-1}}{1-x} dx.$$

Using entry 3.231.5

$$\int_0^1 \frac{x^{\mu-1} - x^{\nu-1}}{1-x} dx = \psi(\nu) - \psi(\mu)$$

it follows that

$$\int_0^1 \frac{x^{n-1} + x^{n-1/2} - 2x^{2n-1}}{1-x} dx = 2\psi(2n) - \psi(n) - \psi(n + 1/2).$$

The identity

$$\psi(2t) = \frac{1}{2} (\psi(t) + \psi(t + 1/2)) + \ln 2$$

gives the result.