## PROOF OF FORMULA 3.311.11

$$\int_0^\infty \frac{e^{px} - e^{qx}}{e^{rx} - e^{sx}}\,dx = \frac{1}{r-s}\left[\psi\left(\frac{r-q}{r-s}\right) - \psi\left(\frac{r-p}{r-s}\right)\right]$$

Write the integral as

$$\int_0^\infty \frac{e^{px} - e^{qx}}{e^{rx} - e^{sx}}\,dx = \int_0^\infty \frac{e^{-(r-p)x} - e^{-(r-q)x}}{1 - e^{-(r-s)x}}\,dx.$$

The change of variables $t = (r-s)x$ gives

$$\int_0^\infty \frac{e^{px} - e^{qx}}{e^{rx} - e^{sx}}\,dx = \frac{1}{r-s}\int_0^\infty \frac{e^{-(r-p)t/(r-s)} - e^{-(r-q)t/(r-s)}}{1 - e^{-t}}\,dt.$$

Formula 3.311.7 now gives the result.