

PROOF OF FORMULA 3.311.12

$$\int_0^{\infty} \frac{a^x - b^x}{c^x - d^x} dx = \frac{1}{(\ln c - \ln d)} \left[\psi \left(\frac{\ln c - \ln b}{\ln c - \ln d} \right) - \psi \left(\frac{\ln c - \ln a}{\ln c - \ln d} \right) \right]$$

Write the integral as

$$\int_0^{\infty} \frac{e^{x \ln a} - e^{x \ln b}}{e^{x \ln c} - e^{x \ln d}} dx$$

and use 3.311.11 in the form

$$\int_0^{\infty} \frac{e^{px} - e^{qx}}{e^{rx} - e^{sx}} dx = \frac{1}{r - s} \left[\psi \left(\frac{r - q}{r - s} \right) - \psi \left(\frac{r - p}{r - s} \right) \right].$$