

PROOF OF FORMULA 3.311.2

$$\int_0^{\infty} \frac{e^{-ax} dx}{1 + e^{-x}} = \beta(a)$$

The β function in the answer is the *incomplete beta function* defined by

$$\beta(a) = \int_0^1 \frac{t^{a-1} dt}{1+t}.$$

The change of variables $t = e^{-x}$ gives directly

$$\int_0^{\infty} \frac{e^{-ax} dx}{1 + e^{-x}} = \int_0^1 \frac{t^{a-1} dt}{1+t}.$$