PROOF OF FORMULA 3.311.4

$$\int_0^\infty \frac{e^{-qx}\,dx}{1-ae^{-px}} = \sum_{k=0}^{\infty} \frac{a^k}{q+kp} = \frac{1}{q}{}_2F_1\left[\frac{q}{p}, 1; 1+\frac{q}{p}; a\right]$$

The expansion

$$\frac{1}{1-ae^{-px}} = \sum_{k=0}^{\infty} a^k e^{-pkx}$$

is integrated term by term to produce the result.