PROOF OF FORMULA 3.311.7

$$\int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{1 - e^{-x}}\, dx = \psi(\nu) - \psi(\mu)$$

Let $t = e^{-x}$ to obtain

$$\int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{1 - e^{-x}}\, dx = \int_0^1 \frac{t^{\mu-1} - t^{\nu-1}}{1 - t}\, dt.$$

The result now follows from the representation

$$\psi(a) = -\int_0^1 \left(\frac{1}{\ln t} + \frac{t^{a-1}}{1 - t} \right) dt.$$