PROOF OF FORMULA 3.311.8

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{b - e^{-x}} = \frac{\pi\ \cot(\pi\mu)}{b^{1-\mu}}$$

The $t = e^{-x}$ to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{b - e^{-x}} = \int_0^{\infty} \frac{t^{\mu-1}\,dt}{b - t}.$$

The change of variables $t = bs$ now gives

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{b - e^{-x}} = b^{\mu-1} \int_0^{\infty} \frac{s^{\mu-1}\,ds}{1 - s}.$$

To evaluate this integral, write

$$\int_0^{\infty} \frac{s^{\mu-1}\,ds}{1 - s} = \int_0^1 \frac{s^{\mu-1}\,ds}{1 - s} + \int_1^{\infty} \frac{s^{\mu-1}\,ds}{1 - s}.$$

The change of variables $s \mapsto 1/s$ in the second integral yields

$$\int_0^{\infty} \frac{s^{\mu-1}\,ds}{1 - s} = \int_0^1 \frac{s^{\mu-1} - s^{-\mu}}{1 - s}\,ds.$$

This is evaluated in 3.231.1 as $\pi\cot(\pi\mu)$.