

PROOF OF FORMULA 3.311.9

$$\int_0^{\infty} \frac{e^{-\mu x} dx}{b + e^{-x}} = \frac{\pi b^{\mu-1}}{\sin \pi \mu}$$

Let  $t = e^{-x}$  to obtain

$$\int_0^{\infty} \frac{e^{-\mu x} dx}{b + e^{-x}} = \int_0^{\infty} \frac{t^{\mu-1} dt}{b + t}.$$

The change of variables  $t = bs$  gives

$$\int_0^{\infty} \frac{e^{-\mu x} dx}{b + e^{-x}} = b^{\mu-1} \int_0^{\infty} \frac{s^{\mu-1} ds}{1 + s}.$$

The integral representation

$$B(u, v) = \int_0^{\infty} \frac{t^{u-1} dt}{(1 + t)^{u+v}},$$

shows that the last integral is

$$B(\mu, 1 - \mu) = \Gamma(\mu)\Gamma(1 - \mu) = \frac{\pi}{\sin \pi \mu}.$$